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Study Guide: Similarity and Dilations

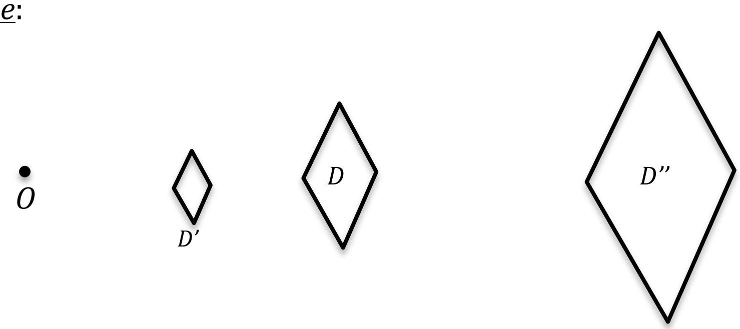
Dilations

A **dilation** is a transformation that moves a point a specific distance from a center of dilation as determined by the scale factor (r).

Properties of Dilations

- Dilations map lines to lines, segments to segments, angles to angles, and rays to rays.
- The length of a line segment dilated from a center is equal to the length of the original line segment multiplied by the scale factor ($|P'Q'| = r|PQ|$).
- The above bullet point means that corresponding line segments are proportional.
- Corresponding angles of dilated objects are congruent.
- A scale factor greater than 1 ($r > 1$)will "push out" the dilated point from the center of dilation.
- A scale factor less than 1 ($r < 1$) will "pull in" the dilated point toward the center of dilation.
- If you dilate a line segment from the same center, the dilated line will either collinear or parallel.

Example:



In this example, the original diamond D is in the middle.
It is dilated from center O by a scale factor of (approx.) $\frac{1}{2}$ to generate D'.
D is dilated from center O by a scale factor of (approx.) 2 to generate D''.

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DILATIONS ON A COORDINATE PLANE

Performing dilations can be easier if you use a coordinate plane.

RULE: If center of dilation is $(0,0)$, and scale factor is r, then the dilation of $A\ (x, y)$ is A', located at (rx, ry).

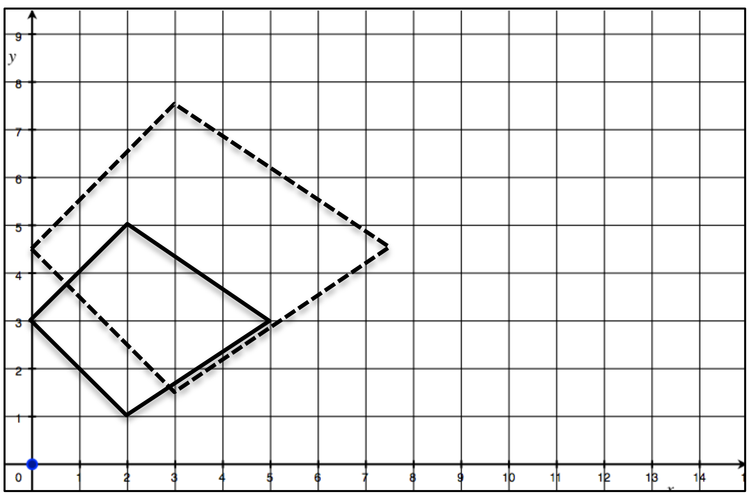
RULE IN WORDS: If your center of dilation is the origin, simply multiply the original coordinates by the scale factor to find the coordinates of the dilated point.

What if your center of dilation is NOT the origin?

In this case, you'll need to measure the horizontal and vertical distance from the center of dilation and multiply by the scale factor to determine the distance of the dilated point from the center of dilation.

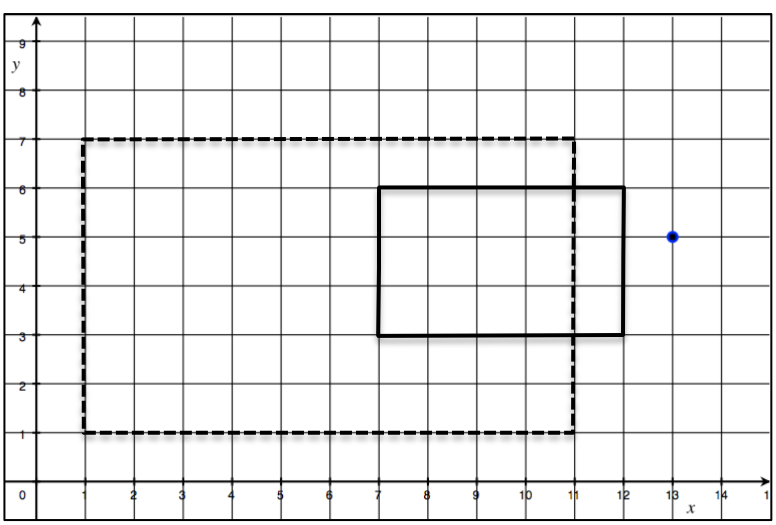
When given two similar shapes, you can locate the center of dilation (and determine the scale factor) by drawing rays that go through all corresponding points. The common point of intersection for all rays is the center of dilation.

Example: Center of dilation at origin:



The solid shape has been dilated by a scale factor of 3/2 from the origin to produce the dashed shape.

Example: Center NOT at the origin:



The solid shape has been dilated by a scale factor of 2 from point $(13, 5)$ to produce the dashed shape.

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Fundamental Theorem of Similarity

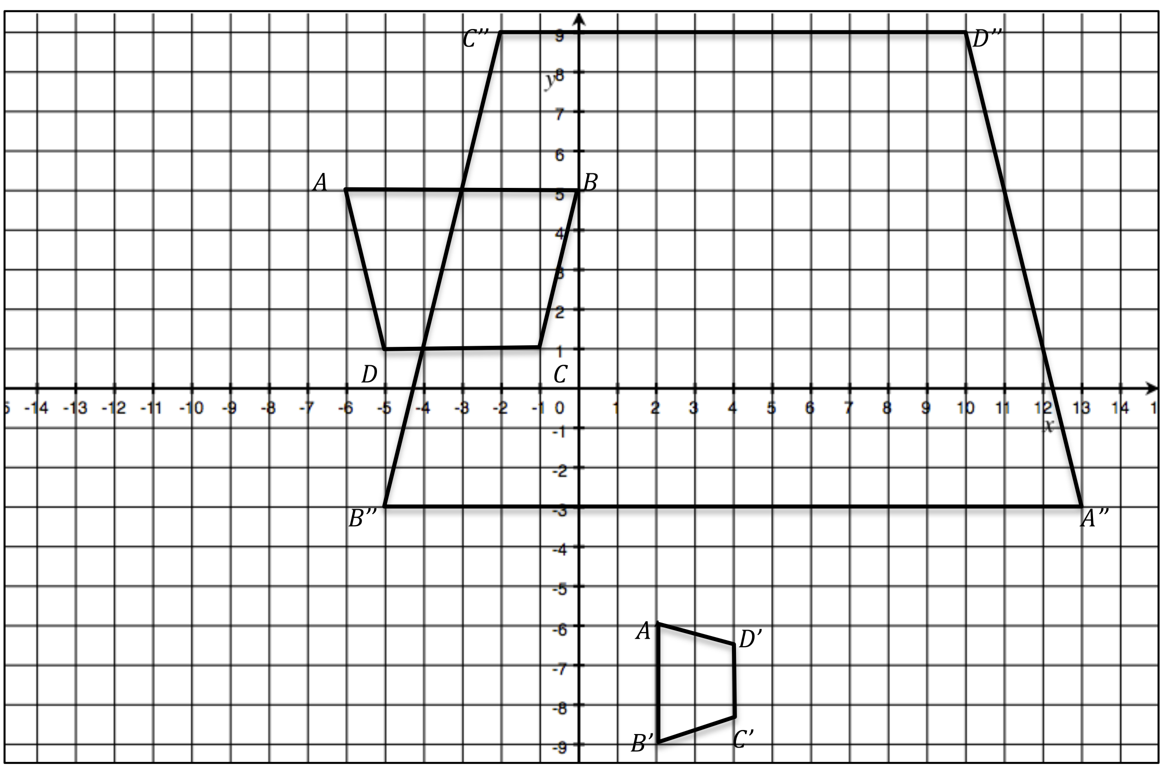
The Fundamental Theorem of Similarity makes two primary conclusions:
1. When you dilate points P and Q from a center O, and the three points are NOT collinear, then the line segment PQ and the dilated line segment $P'Q'$ are parallel.

2. Furthermore, the length of the $P'Q'$ is equal to the length of PQ multiplied by the scale factor. In symbols: $|P'Q'| = r|PQ|$.

Similarity thru rigid motions

Dilations alone might not be enough to prove that two shapes are similar. Often times, you'll need to perform a rotation or reflection first before doing your dilation. Pay close attention to the orientation AND coordinate points of corresponding shapes when trying to determine the sequence of rigid motions.

Example:



To dilate from *ABCD* to either *A'B'C'D'* or *A"B"C"D"*, we need a sequence of rigid motions AND dilations.

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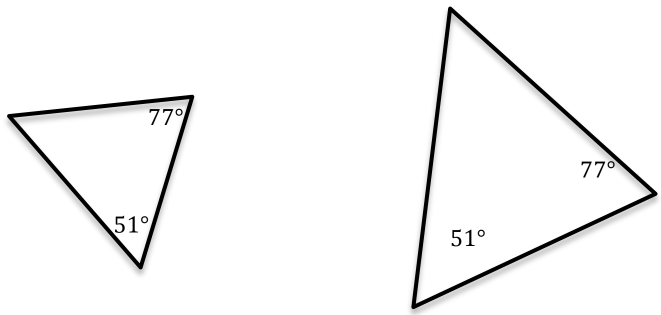
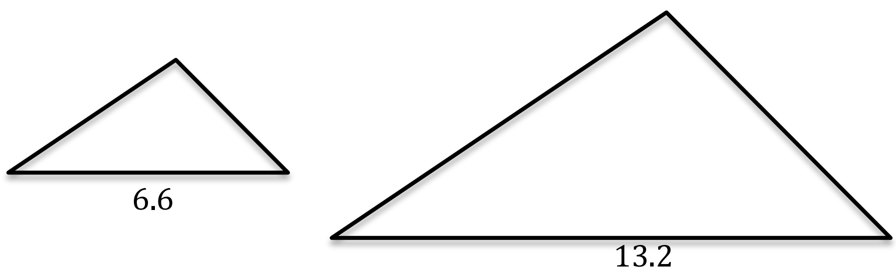
WAYS TO PROVE TRIANGLES SIMILAR

You can prove that two triangles can be similar to each other by showing that:

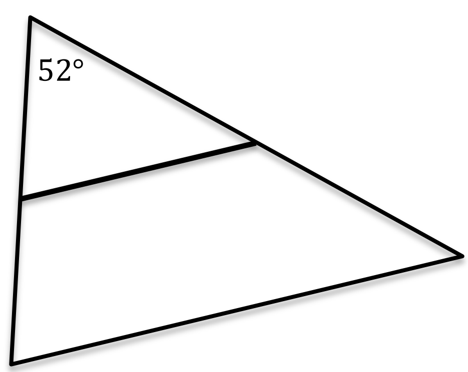
- **Angle-Angle Similarity**: When two pairs of corresponding angle measures are equal, two triangles are similar.



- If the lengths of corresponding side are proportional, two triangles are similar



- S-A-S similarity. If a corresponding angle is congruent and the two sides that form that angle have a directly proportional relationship, the triangles are similar.



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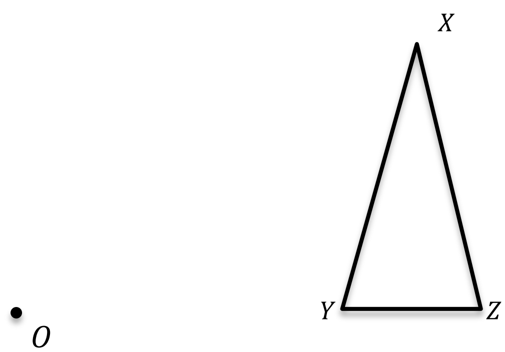
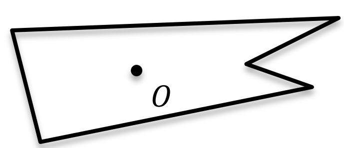
PROBLEM SET

(Complete these problems before you submit your assessment for grading).
Dilate each object based on the directions.

Dilate from Center O by a Scale Factor of $\frac{1}{2}$ and then a Scale Factor of 2.



Dilate from Center O by a Scale Factor of 3.



Briefly describe the process for dilating by a scale factor of 2 using a compass.

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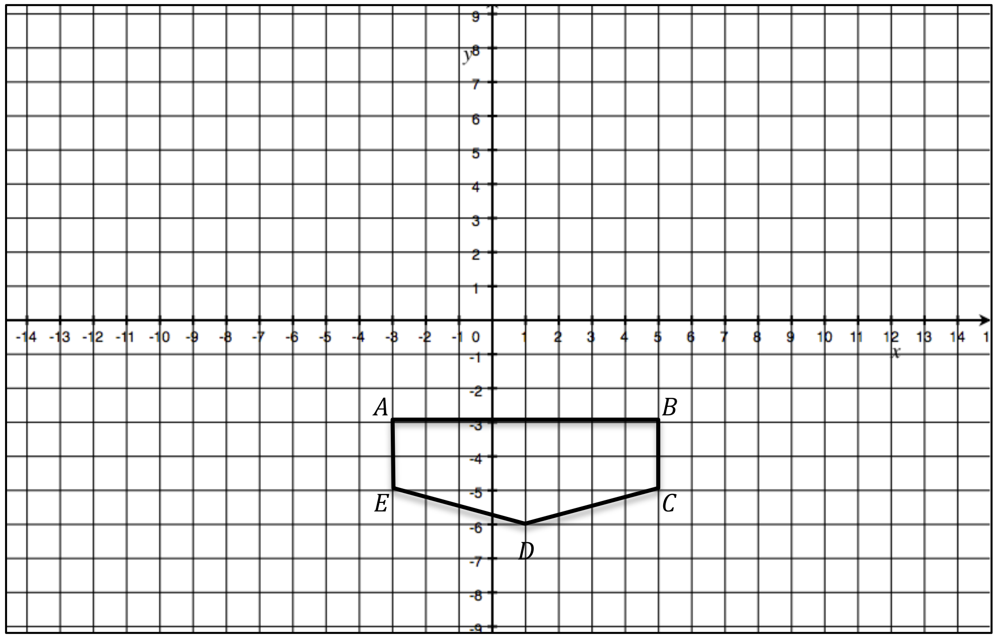
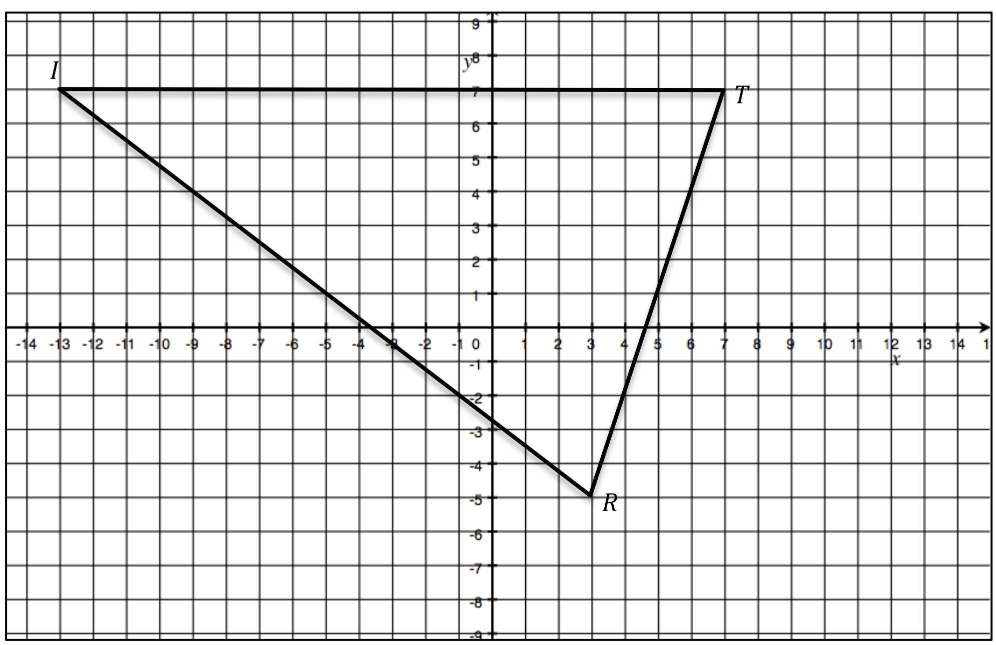
Dilate according the instructions.

Dilate the shape from the center of dilation at (2, -8) by scale factors of 2 and 3.



Dilate ΔTRI from the center of dilation at $(-9, -1)$ by scale factors of $\frac{1}{4}$ and $\frac{1}{2}$.



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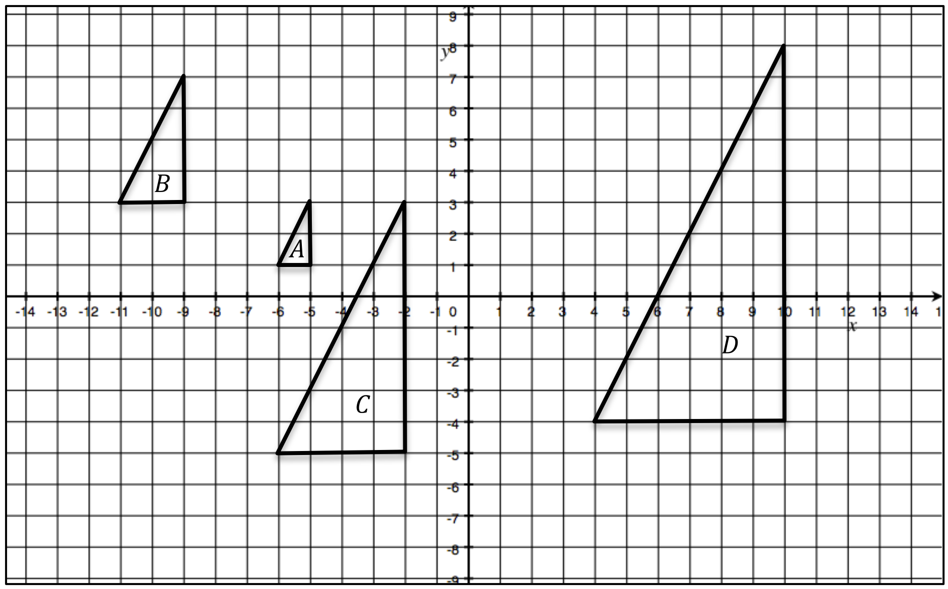
Shapes A, B, C, and D are similar. Identify at least 4 dilations, noting centers of dilation and scale factors for each dilation.



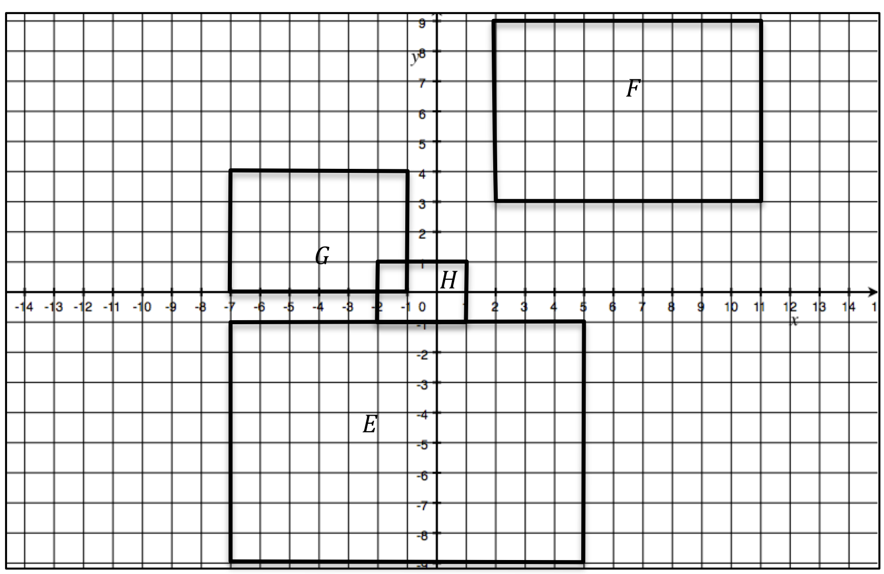
1.

2.

3.

4.

Shapes E, F, G, and H are similar. Identify at least 4 dilations, noting centers of dilation and scale factors for each dilation.



1.

2.

3.

4.

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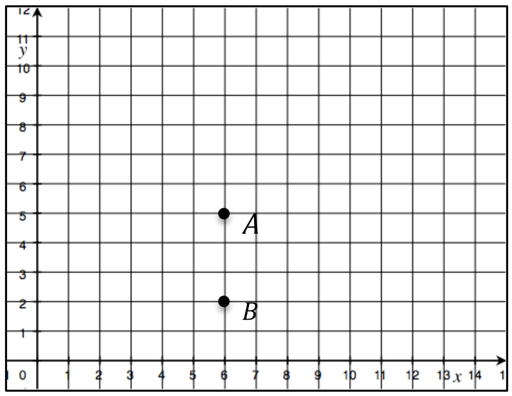
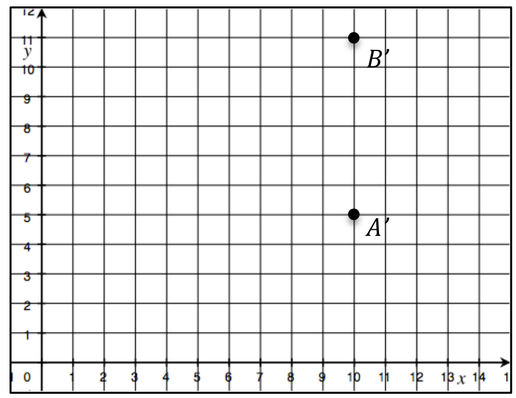
Dilate the following

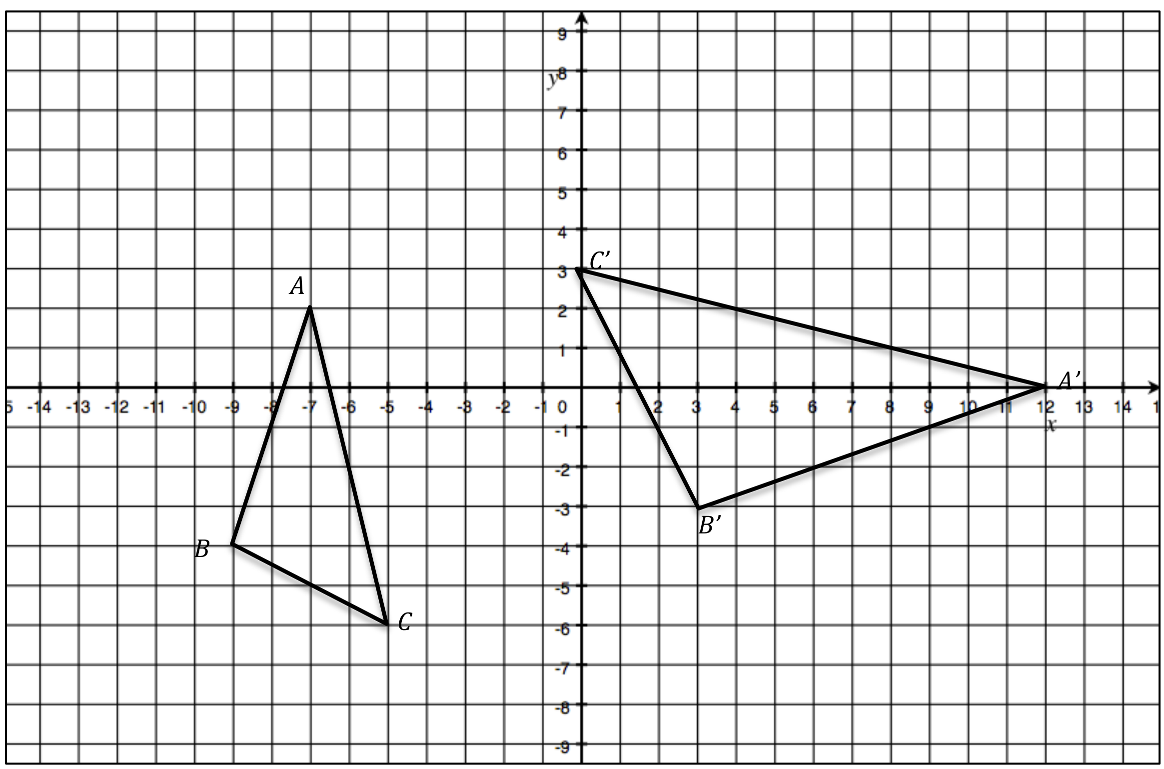
Find A' and B' when A and B are dilated from the origin by a scale factor of $\frac{9}{4}$. Also, find the length of $\overline{A'B'}$.

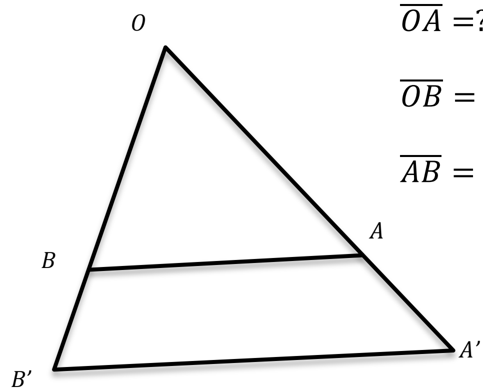


A' and B' are dilations from the origin with a scale factor of $\frac{4}{5}$. Find A and B and the find the length of $\overline{AB}$.



Describe the sequence of transformations that would map one shape to the other.





$\overline{OA}$ =? $\overline{OA'} = 14$

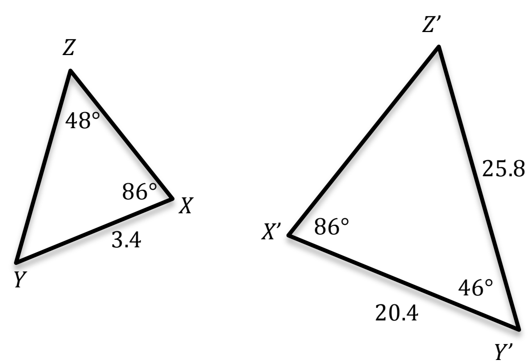
$\overline{OB} = 3.2$ $\overline{OB'} = ?$

$\overline{AB} = 5$ $\overline{A'B'} = 17.5$

1. Suppose $\overline{AB}$ and $\overline{A'B'}$ are parallel. Is ΔOAB similar to $\Delta OA'B'$? Explain.

2. What is the length of $\overline{OB'}$?

3. What is the length of $\overline{OA}$?



1. Is ΔXYZ similar to $\Delta X'Y'Z'$. Explain.

2. Find the length of $\overline{YZ}$.