

STUDY GUIDE: SLOPE AND LINEAR EQUATIONS

RATES AND UNIT RATES

A **rate** is a proportional relationship between two quantities.

Unit rate is a rate where the second quantity is 1.

Example: Pauline can mow 35 square feet of lawn in 2.5 minutes. (this is a rate). She can mow 14 square feet per minute (this is a unit rate).

We set up proportions to find unit rates and write linear equations in 2 variables.

We solve for y .

Example:

$$\frac{y}{x} = \frac{35}{2.5}$$

Set up proportion

$$2.5y = 35x$$

Multiply the numerators by the denominator of the other fraction.

$$y = 14x$$

Solve for y .

Rates are **directly proportional** if their graph lies on a straight line AND passes through the origin.

SLOPE

Slope is a measure of steepness or slant of a line. It is also the measure of the rate of the change of a line.

Slope can be expressed as a unit rate: How many vertical units does a line move for every one horizontal unit?

The **slope formula** states that:

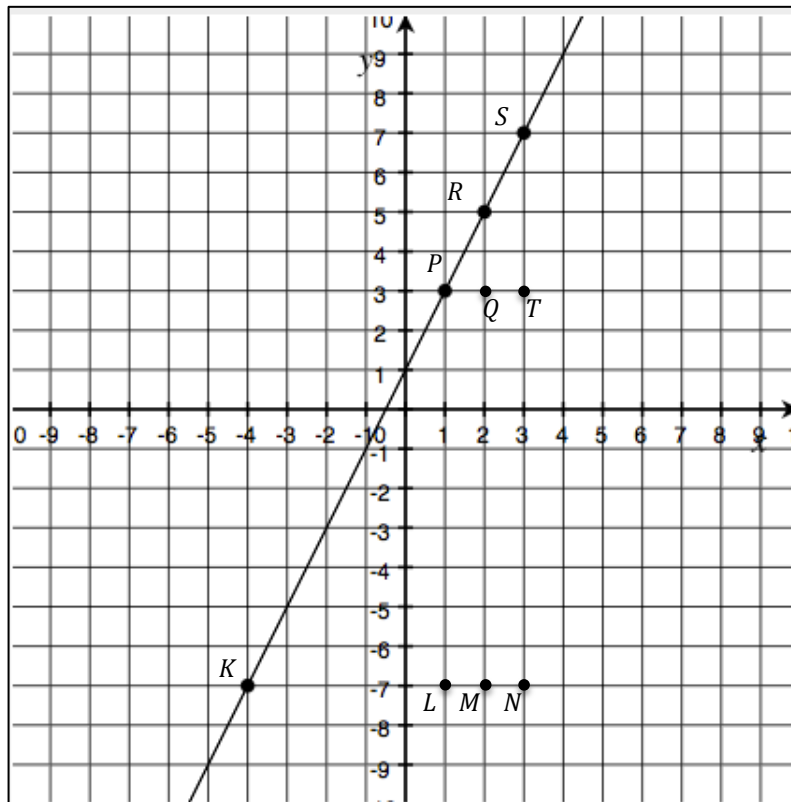
$$m = \frac{(p_2 - r_2)}{(p_1 - r_1)} = \frac{\text{rise}}{\text{run}} = \frac{\text{difference in } y \text{ - values}}{\text{difference in } x \text{ - values}}$$

Where the coordinates of point P on a line are represented by $P = (p_1, p_2)$ and the coordinates of point R on the line are represented by $R = (r_1, r_2)$.

SLOPE TRIANGLES

We can use the concept of **slope triangles** to prove the slope of a line is the same between any two points on the line:

- Show AA similarity of two triangles.
- **Know that corresponding sides of similar triangles are proportional...**and this means that the slope of a line is the same for the entire distance of the line.



If you create several different right triangles using the points above, you can see that all of the triangles are similar triangles using the concept of AA similarity.

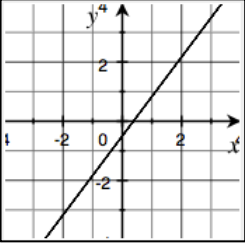
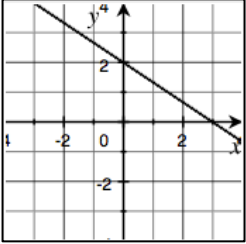
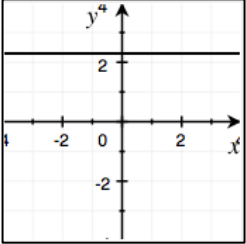
Example: $\triangle PLK \sim \triangle RMK$ because they share a common angle AND they each have right angles...hence, they meet the AA criterion for similarity. This means the proportions of corresponding sides of each triangle are equal.

Example: The proportions of the corresponding sides of the triangles mentioned above are equal...

$$\frac{\overline{RM}}{\overline{MK}} = \frac{\overline{PL}}{\overline{LK}}$$

$$\frac{12}{6} = \frac{10}{5}$$

SLOPE (CONT'D)

Positive Slope Left-to-right inclining lines	Negative slope Left-to-right declining lines	Zero slope Horizontal Lines
		

The greater the absolute value of the slope, the steeper the line!!

Horizontal lines have zero slope, and are written as $y = c$.

Vertical lines have an undefined slope and are written as $x = c$.

EQUATIONS OF LINES

Standard Form of an equation

$ax + by = c$ where a, b , and c are all integers and $a > 0$

We can graph a line knowing the standard form of a linear equation by identifying the x -intercept and y -intercept.

Slope Intercept form of an equation

$y = mx + b$ where m is the slope and b is the y -intercept.

We can graph a line knowing the slope-intercept form of a linear equation by identifying the y -intercept and using the slope to graph another point.

Minimum requirements for finding the equation of a line:

1. Slope and one point on the line (often the one point will be the y -intercept)
2. Two points on the line. (this can be the x -intercept and y -intercept)

Convert from slope-intercept form to standard form:

- Gather ax and by terms on left side of the equation.
- Make sure that a, b , and c are all integers and a is positive.

Convert from standard form to slope-intercept form:

- Solve for y .

PROBLEM SET

Complete the problems below before taking our test. If you have any questions, be sure to ask either at a review session.

A phone company has two plans for data that are as follows:

Plan A includes an up front service charge of \$45 per month, and charges \$2.50 for each GB of data used.

Plan B includes no service charge, and charges \$7.50 for each GB of data.

1. Write equations that represent each cell phone plan, where y is the cost of the data, and x is the amount of data (in GB). Identify the slope and explain what it means.

Plan A: $y = 2.5x + 45$

Plan B: $y = 7.5x$

2. Graph the equations on the coordinate plane.

See coordinate plane.

3. When is it better for you to go with plan A? When does it make sense to go with plan B?

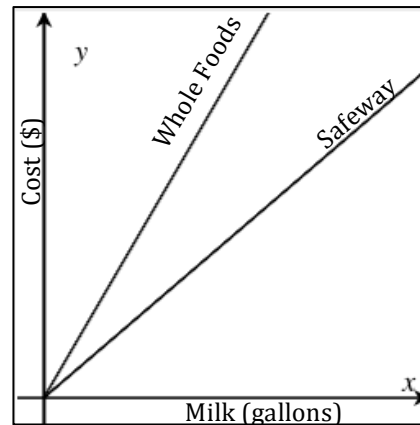
If you're planning to use **more** than 9GB of data, I'd go with plan A, but less than 9GB of data, go with plan B. the break even cost is \$67.50.



Sophia and Colby were given these two points $(2, -5)$ and $(-3, 5)$, and told to write the equation of the line. Sophia thought the equation was $y = -2x - 1$. Colby thought she was wrong and that the equation was $y = -\frac{1}{2}x - 4$. Who's right? Why? What did the other person do wrong?

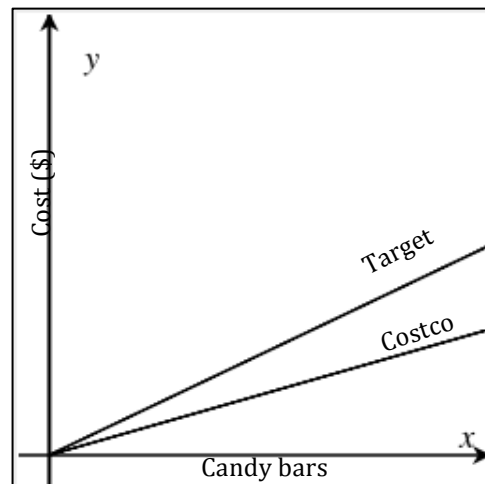
Sophia is correct. She computed the slope as $\frac{\text{change in } y}{\text{change in } x}$ and found that $\frac{5 - (-5)}{-3 - 2} = -2$. She then found that the y-intercept was -1 by substituting in a point and the slope in the $y = mx + b$ formula. It's likely that Colby confused $\frac{\text{rise}}{\text{run}}$ and instead calculated the slope as $\frac{\text{run}}{\text{rise}}$ incorrectly.

A gallon of milk at Whole Foods cost \$6. The graph below compares the cost of milk at Whole Foods and Safeway. Estimate the cost of a half-gallon of milk at Safeway. Justify your reasoning.



The slope of the line representing the cost of milk at Safeway is less steep than that of Whole Foods milk, indicating it's less expensive. It looks like the SFY line is about half as steep as the WFM line, which would lead me to believe the price of a half gallon of milk is \$1.50...this is because a GALLON of milk at Whole Foods is \$6, and if this is to be linear, the half gallon would be \$3, and half of \$3 is \$1.50. got it?? Also, if you can find milk at \$1.50 for a half gallon, I'd stock up—that's a good price these days.

12 candy bars at Target cost \$15.00. The graph below compares the cost of candy bars at Target and Costco. Estimate the cost of 1 candy bar at Costco. Justify your reasoning.



The candy bars at Target cost MORE than the candy bars at Costco. I know because the line depicting the prices at TGT are steeper than the of the line depicting prices at COST, and the slope is greater. The unit cost of candy bars at TGT is \$1.25 ($\$15/12$). The COST line looks to be about $2/3$ the steepness of the TGT line, so I'd estimate the COST candy bars to be about \$0.83 each...but if you're thinking \$0.75, you can say you're good too. \$1.00 is too high, and \$0.63 is too low.

In addition to these four problems, the following exercises have been assigned in Khan Academy. Please check the box to indicate that you have either completed 5 in a row or 8 out of 10.

- Graph from slope-intercept equation
- Slope-intercept equation from graph
- Converting Between Slope-Intercept and Standard Form

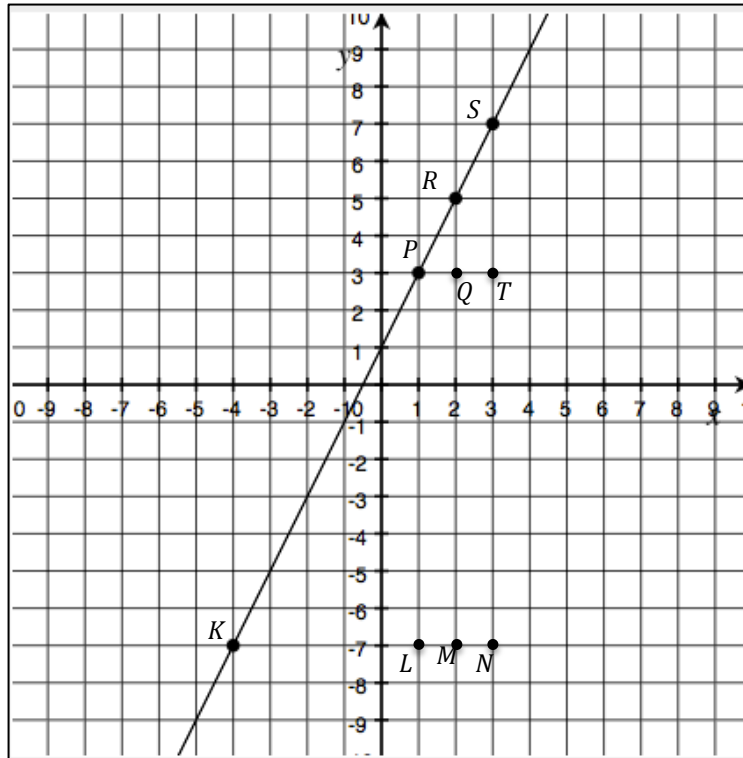
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Use the graph below to complete the questions.



On page 2, we identified two similar triangles ($\triangle PLK$ and $\triangle RMK$). Identify 3 other similar triangles, and show that they are similar by looking at the proportions of the corresponding sides.

3 triangles: $\triangle STP$, $\triangle SNK$, $\triangle RQP$

Proportions of corresponding sides (see page 2 for an example):

$$\frac{ST}{TP} = \frac{SN}{NK} = \frac{RQ}{QP} \quad \frac{4}{2} = \frac{14}{7} = \frac{2}{1} = \text{slope of 2}$$

What is the proportion of the corresponding sides also called? (if you consider the corresponding vertical sides in the numerator and the corresponding horizontal sides in the denominator). **This is the slope.** Once we can prove similarity between a pair of slope triangles (and you should be able to prove this on a test), we can state positively that the proportion of the corresponding sides are equal...also, that proportion measuring the change in vertical distance over the change in horizontal

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distance is called slope, and this is proof that the slope of a line is the same between any two points on the line. YOU WILL NEED TO KNOW THIS!!