

STUDY GUIDE: SYSTEMS OF LINEAR EQUATIONS

SYSTEMS OF LINEAR EQUATIONS

A **system of linear equations** is when two or more linear equations are involved in the same problem.

The **solution for a system of linear equations** is the ordered pair (x, y) that makes BOTH equations true.

Solutions to systems of linear equations: Systems of equations either have one unique solution, no solution, or infinitely many solutions.

One Unique Solution: If two linear equations have *different slopes*, there will be one unique solution, which is at the point of intersection.

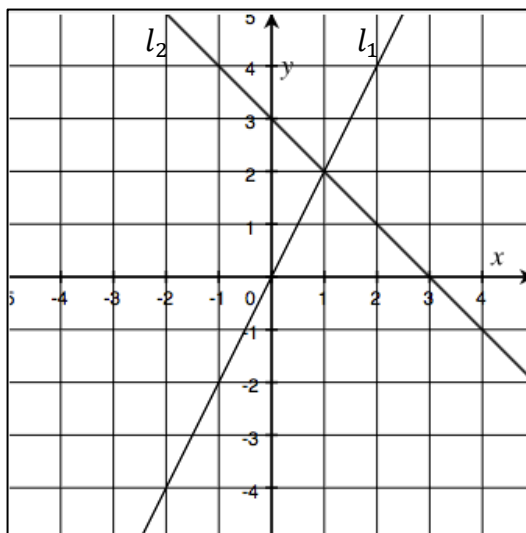
No Solutions: If two linear equations have the *same slope, but different y-intercepts*, the lines are parallel and there is NO solution to the system of equation because there is no point of intersection.

Infinitely Many Solutions: If two linear equations have *the same slope AND the same y-intercept*, the two lines are identical and the system will have infinitely many solutions.

SOLVING SYSTEMS OF EQUATIONS: GRAPHING

On a coordinate plane, the intersection (coordinate pair) of two lines is the solution to a system of linear equations.

Example: The solution to the system of equations below is $(1, 2)$



The equation for l_1 is $y = 2x$. Substitute $(1,2)$ into the linear equation to make sure that the point IS on the line.

The equation for l_2 is $y = -x + 3$. Substitute $(1,2)$ into the linear equation to make sure that the point IS on the line.

SOLVING SYSTEMS OF EQUATIONS: SUBSTITUTION

Systems of equations can be solved by **substituting** an expression from one equation into the other equation.

Examples:

<p>1.</p> $\begin{cases} y = \frac{3}{2}x - 4 \\ y = -x + 9 \end{cases}$ <p>Since y is equal to both $(\frac{3}{2}x - 4)$ AND $(-x + 9)$, those two expressions are equal to each other...</p> $\frac{3}{2}x - 4 = -x + 9$ <p>To solve the system, solve for x, and then substitute your solution for x into one of your original equations and solve for y.</p>	<p>2.</p> $\begin{cases} y = 2x - 1 \\ 3x + 2y = 6 \end{cases}$ <p>Since y is isolated in the first equation, we can substitute "$2x - 1$" wherever we see y in the second equation:</p> $3x + 2(2x - 1) = 6$ <p>To solve this system, we can solve for x, and then substitute your solution for x into one of the original equations and solve for y.</p>
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WHEN TO SOLVE USING SUBSTITUTION:

1. When both equations are already in slope intercept form (as in example 1 above)
2. When one variable is isolated in one of the linear equations (as in example 2 above)

SOLVING SYSTEMS OF EQUATION: ELIMINATION

Solve systems of linear equations using **elimination** by **ADDING** equations to eliminate a variable. **NOTE:** You might need to multiply an **ENTIRE** equation before you can add the two equations to eliminate a variable.

Examples:

$\begin{cases} 6x - 5y = 21 \\ 2x + 5y = -5 \end{cases}$ <p>Since $5y$ and $-5y$ are opposites, if we add the two equations together, we can eliminate the y-variable.</p> $8x + 0y = 16$ <p>From here, solve for x, and then substitute your solution for x into one of your original equations and solve for y.</p>	$\begin{cases} 6x - 4y = 18 \\ 2x + 3y = 4 \end{cases}$ <p>Since there are no opposites, multiply the second equation by -3 (to create a situation where you have opposites ($6x$ and $-6x$)) and then follow the steps in the example on the left.</p> $\begin{cases} 6x - 4y = 18 \\ -6x - 9y = -12 \end{cases}$
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THE PROBLEM SET

Complete the problems below and turn this sheet in prior to completing the assessment

Determine whether each of the following have one unique solution, infinitely many solutions or no solutions. Do NOT solve the system, but explain HOW YOU KNOW the number of solutions in one sentence.

$$\begin{cases} y = \frac{3}{4}x + 12 \\ 3x - 4y = 48 \end{cases}$$

No solution—slope same, but y-intercept different—lines are parallel

$$\begin{cases} 4x - 2y = -7 \\ y = \frac{1}{2}x - 14 \end{cases}$$

No solution—same slope, but y-intercept different—lines are parallel

$$\begin{cases} x = 3y - 4 \\ 2x + 6y = 12 \end{cases}$$

One solution—different slopes. Lines intersect only once.

$$\begin{cases} 7y = 14x - 21 \\ 4x - 2y = 6 \end{cases}$$

Infinitely many solutions—same slope and same y-intercept. These are the same lines.

Solve the following using **EITHER** substitution or elimination—your choice. Whichever you choose, write a sentence explaining WHY you selected the method you did.

$$\begin{cases} y = 2x - 1 \\ x + 3y = 9 \end{cases}$$

Solve by substitution because y is already isolated in the top equation.
Solution: $(\frac{12}{7}, \frac{17}{7})$

$$\begin{cases} 2x - 5y = 9 \\ 4x + 2y = 3 \end{cases}$$

Solve by elimination. Multiply top equation by 2 (or -2) and subtract (or add) equations.
Solution: $(\frac{11}{8}, -\frac{5}{4})$

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$\begin{cases} y = x - 4 \\ x + y = 4 \end{cases}$ <p>Solve by substitution because y is already isolated in the top equation. Solution: (4,0)</p>	$\begin{cases} x + 3y = 14 \\ 4x - 6y = 2 \end{cases}$ <p>You could solve this either way— substitute by isolating x in the top equation in one step or eliminate by multiplying the top equation by 2 and adding (or multiply by 4 and subtract)...the bottom line is that solutions may vary Solution: (5,3)</p>
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<p>Solve the following word problems.</p>	
<p>Bananas have 105 calories each. Apples have 75 calories each. In one day, Mary only eats those two fruits, and she eats 13 pieces of fruit, and consumes a total of 1215 calories. How many of each fruit did Mary consume? How do you think her stomach feels?</p> <p>Mary has 8 bananas and 5 apples and her stomach feels great!</p>	<p>On iTunes, you can download songs for \$1.29, and rent HD movies for \$5.99 each. If Dominic spends \$63.40 for 20 total pieces of media (this includes both movies and songs), how many movies did he rent? How many songs did he download?</p> <p>Dominic downloaded 12 songs and 8 videos</p>

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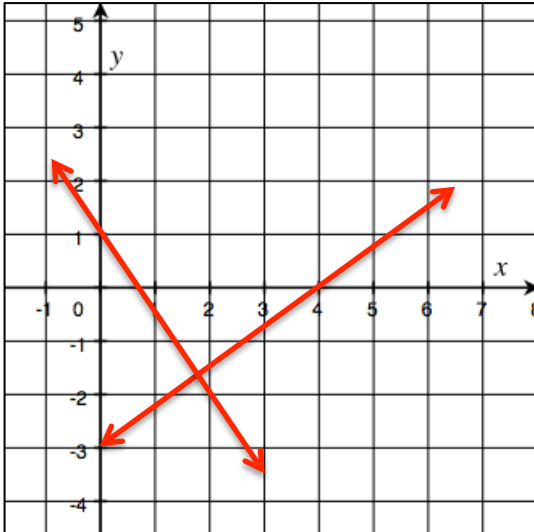
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Solve the following systems of equations by graphing, using substitution, and using elimination.

$$\begin{cases} y = -\frac{3}{2}x + 1 \\ 3x - 4y = 12 \end{cases}$$



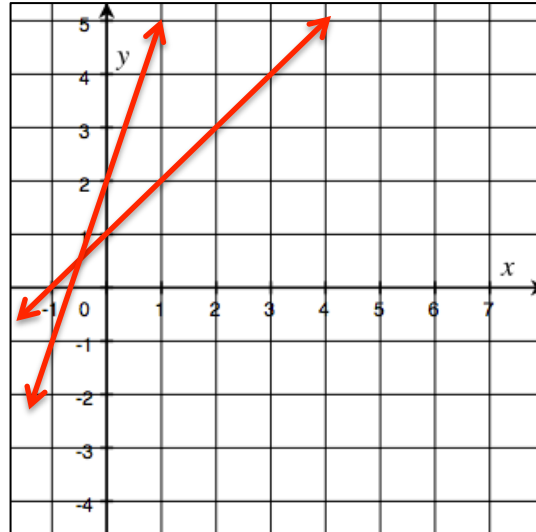
Solve by substitution:

Solution for both:

$$\left(\frac{16}{9}, -\frac{5}{3}\right)$$

Solve by elimination:

$$\begin{cases} x - y = -1 \\ y = 3x + 2 \end{cases}$$



Solve by substitution:

Solution for both:

$$\left(-\frac{1}{2}, \frac{1}{2}\right)$$

Solve by elimination: