

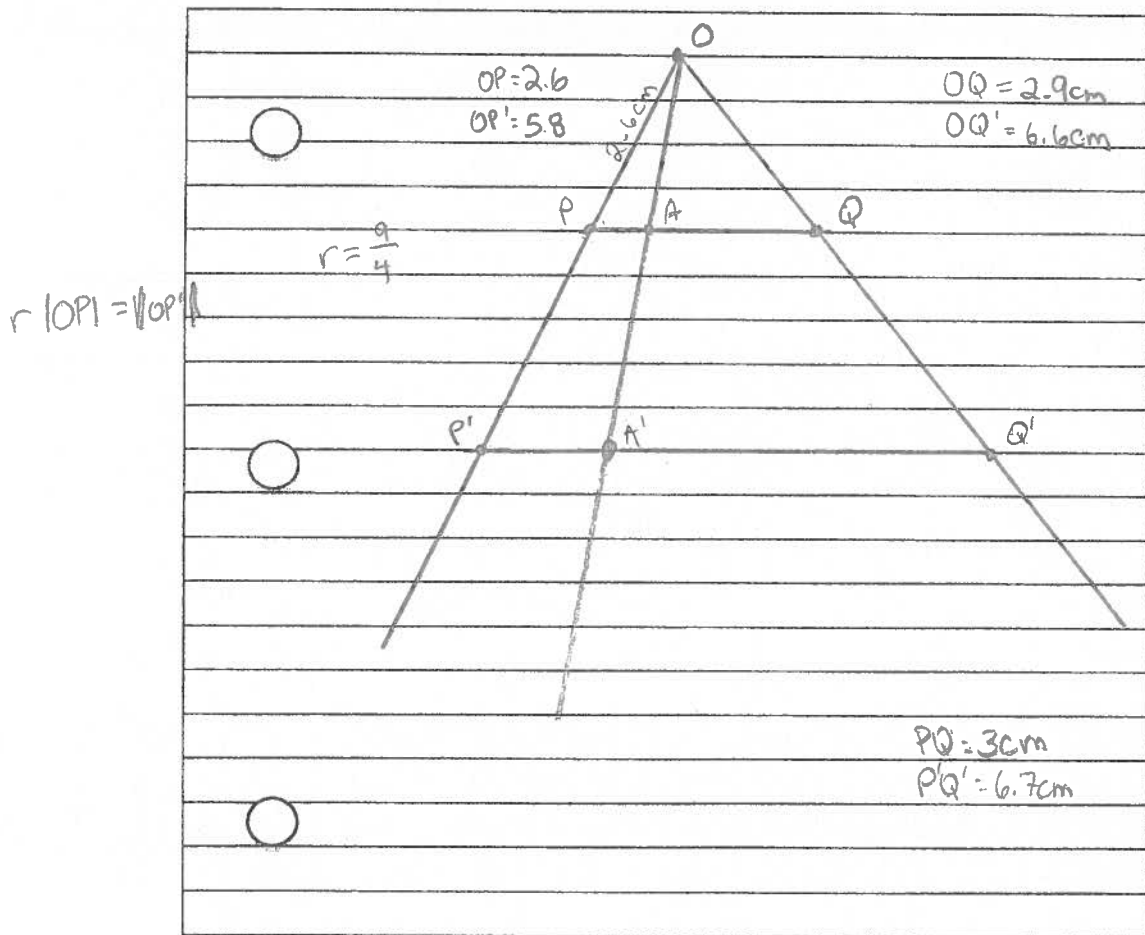
**LEARNING OBJECTIVE:**

We will verify the Fundamental Theorem of Similarity (G8M3L4)

**CONCEPT DEVELOPMENT:**

**The Fundamental Theorem of Similarity (FTS):** Given a dilation with center  $O$  and scale factor  $r$ , then for any two points  $P$  and  $Q$  in the plane so that  $O, P,$  and  $Q$  are not collinear, the lines  $PQ$  and  $P'Q'$  are parallel, where  $P' = \text{dilation}(P)$  and  $Q' = \text{dilation}(Q)$  and furthermore  $|P'Q'| = r|PQ|$ .

**"Lined Paper" Proof**



1. On line 2, make center  $O$
2. Mark point  $P$  4 rows down
3. Draw ray  $\vec{OP}$
4. Mark  $P'$  9 rows down from  $O$
5. Find the scale factor.
6. Mark point  $Q$  4 rows down from  $O$  and draw ray  $\vec{OQ}$
7. Mark  $Q'$  using the same scale fac
8. Connect  $PQ$  and connect  $P'Q'$
9. Measure  $OP$  &  $OP'$
10. Measure  $OQ$  &  $OQ'$

11. Measure  $PQ$  and  $P'Q'$
12. Draw another ray from  $O$ .
13. Label  $A$  &  $A'$

$$|OA| \cdot \frac{9}{4} = |OA'|$$

$$\frac{|P'A'|}{|PA|} = \frac{9}{4}$$

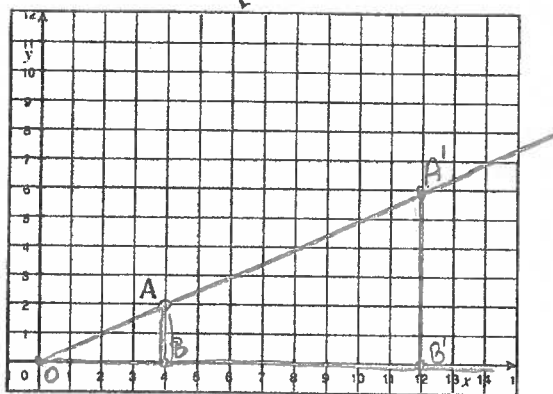
VERIFY THAT THEY HAVE THE SAME SCALE FACTOR

**GUIDED PRACTICE:**

**How to use the FTS to Determine Points of Dilation on a Coordinate Plane**

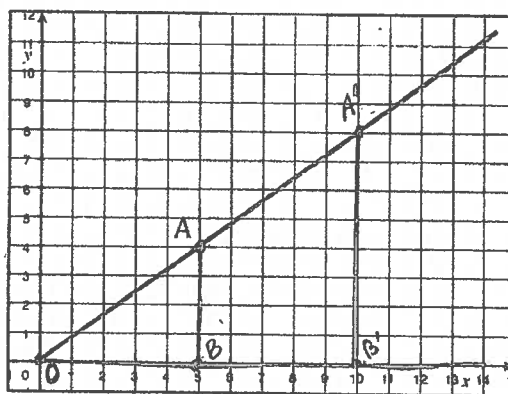
1. Locate the center of dilation (for this lesson, it will be the origin).
2. Locate the original point that is being dilated. Draw a ray from the origin through the point and label the point A.
3. Draw a vertical line from Point A to the x-axis, and label the point of intersection with the x-axis B.
4. Find B' by multiplying the distance of segment OB by the given scale factor.
5. Draw a vertical line from B' to the ray OA. The point of intersection with your ray is the dilation of A.

Find A' when A is dilated from the origin by a scale factor of 3.

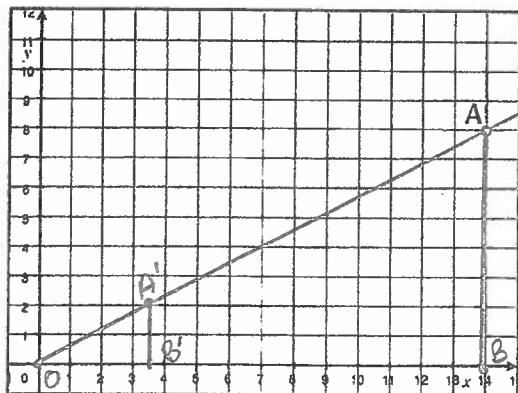


$A' (12, 6)$

Find A' when A is dilated from the origin by a scale factor of 2.

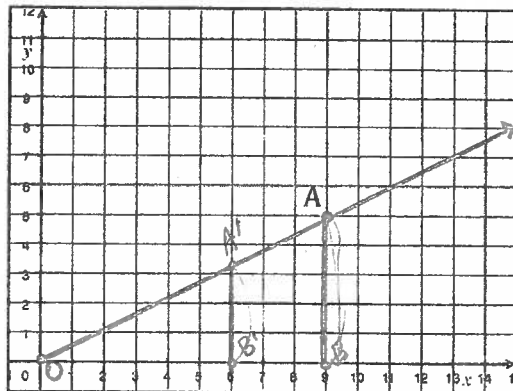


Find A' when A is dilated from the origin by a scale factor of  $\frac{1}{4}$ .



$A' = (3.5, 2)$

Find A' when A is dilated from the origin by a scale factor of  $\frac{2}{3}$ .

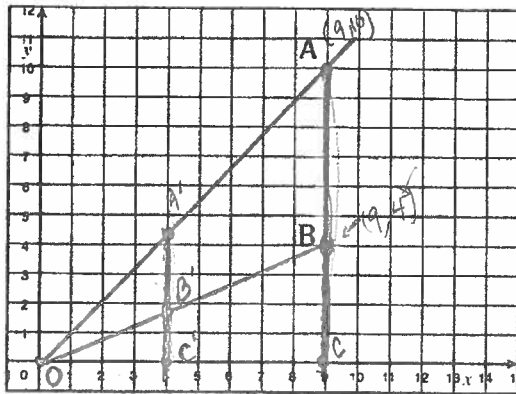


$A' (6, \frac{10}{3})$

Mr. Rogove

Date: \_\_\_\_\_

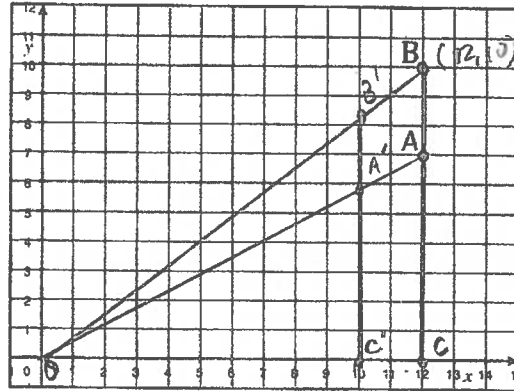
Find  $A'$  and  $B'$  when  $A$  and  $B$  are dilated from the origin by a scale factor of  $\frac{4}{9}$



$$A' \left( 4, \frac{40}{9} \right) = \left( \frac{4}{9} \cdot 9, \frac{4}{9} \cdot 10 \right)$$

$$B' \left( 4, \frac{16}{9} \right) = \left( \frac{4}{9} \cdot 9, \frac{4}{9} \cdot 4 \right)$$

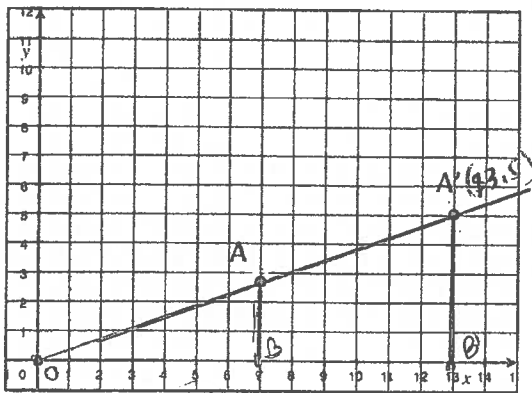
Find  $A'$  and  $B'$  when  $A$  and  $B$  are dilated from the origin by a scale factor of  $\frac{5}{6}$



$$A' \left( 10, \frac{35}{6} \right)$$

$$B' \left( 10, \frac{25}{3} \right)$$

Find the scale factor of the dilation from  $A$  to  $A'$ .

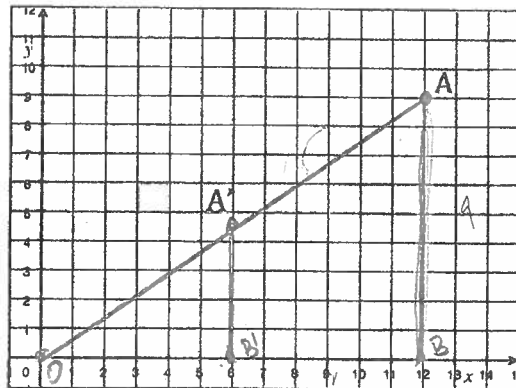


$$r = \frac{|OA'|}{|OA|} = \frac{|OB'|}{|OB|} = \frac{|A'B'|}{|AB|}$$

$$r = \frac{13}{7}$$

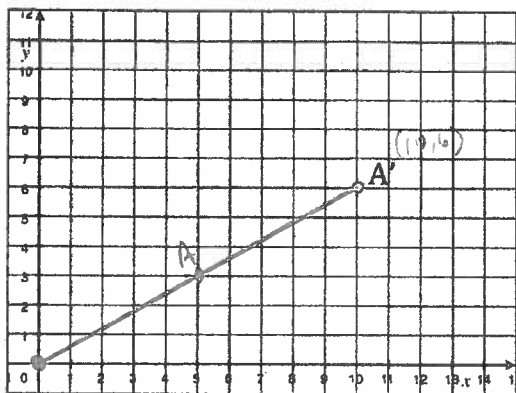
$$A \left( 7, \frac{35}{13} \right)$$

Find the scale factor of the dilation from  $A$  to  $A'$ .



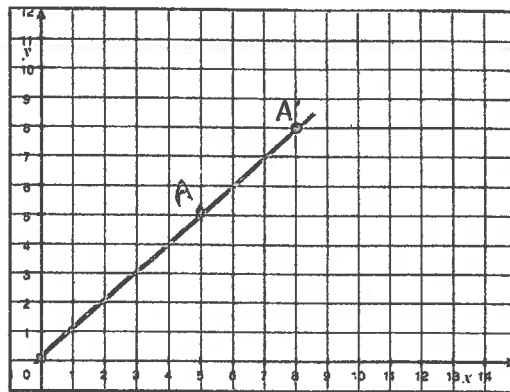
$$\frac{|OB'|}{|OB|} = \frac{6}{12} = \frac{1}{2} = r$$

Find  $A$  when  $A'$  has been dilated from the origin by a scale factor of 2.



$$A = (5,3)$$

Find  $A$  when  $A'$  has been dilated from the origin by a scale factor of  $\frac{8}{5}$ .



$$A = (5,5)$$

### Line of Learning:

What have you learned about the concepts of similarity in general and dilations specifically?

- When you dilate on a coordinate plane, your dilations may not have integer coordinates.
- IF  $r > 1$ , the dilation expands the point  
IF  $r < 1$ , the dilation shrinks the point.
- Express coordinate points as improper fractions (or decimals)
7. IF you dilate a shape by scale factor of  $r$ , the area of new shape is  $A \cdot r^2$
- The scale factor tells us how much to multiply by.
- Similarity <sub>n</sub> shows a proportional relationship.
- \* IF we dilate 2 points from the same center by the same scale factor and connect the points, the resulting lines will be parallel.