Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

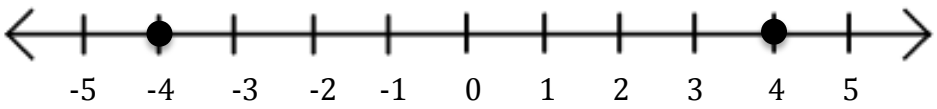
Solutions to Equations and Inequalities Study Guide

Solutions to Equations

Solutions to Equations are expressed in 3 ways:

<u>In Words:</u> the equation $x^2 = 16$ has solutions of 4 and -4. Or, $x^2 = 16$ is a true statement when $x = 4$ or $x = -4$.

<u>Graphically on a Number line:</u>



Use solid dots to indicate points on the number line that are included in the solution set and open dots to indicate points to be excluded.

Solid lines are used to indicate intervals of points on the number line (all real numbers) that are to be included in the set.

<u>Using Set notation:</u> $\{4, -4\}$ is the set containing 4 and -4 and is the solution set to the equation listed above.

Sometimes it might be difficult to list the elements of a solution set easily. In those cases, you the following notation:

{variable symbol number type| description}

<u>Example</u>:

$$\{x\ real\ |x > 0\}$$

is the set of all real numbers that are greater than 0.

The vertical bar means "such that…"

The null or empty set is denoted by any of the following:

$\emptyset$ or $\{\ \}$

(For more information on this, especially set notation, please refer to Lesson 85 or pages s.54-s.69 from the Algebra 1 Module 1 workbook you received)

Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

STEPS USED IN SOLVING EQUATIONS

Solving equations is nothing more than performing a series of moves that use various properties of mathematics to manipulate the equation in order to isolate your variable.

***You are trying to find out the values for the variable that make the equation true.**

Properties of Equality: These properties apply to the operations of addition, subtraction, multiplication, and division. If you perform an operation to one side of an equation, you must perform it to the other side of the equation in order to maintain equality.

Examples:

- The equation $(x+1)(x-6)=3x+34$ will have the same solution set as the equation $(x+1)(x-6)-12=3x+22$ because of the **subtraction property of equality**.

- The equation $(3x-4)(x+19)=4x+23$ will have the same solution set as the equation $2(3x-4)(x+19)=8x+46$ because of the **multiplication property of equality.**

Distributive Property: This does NOT change the value of an expression, and will not affect the solution to an equation. The distributive property allows you to rewrite expressions using multiplication and addition (or subtraction).

Example:

$$3(x+13)=3x+39$$

***Non-example*:**

$$\mathbf{4(3xy) \neq 12 \cdot 4x \cdot 4y}$$

Commutative Property: This does NOT change the value of an expression, and will not affect the solution to an equation. The commutative property allows you to rewrite expressions involving addition or multiplication in any order.

Example: (addition)

$$3x+4y-x=3x-x+4y$$

Example: (multiplication)

$$4c \cdot 3d = 4 \cdot 3 \cdot c \cdot d$$

Associative Property: This does NOT change the value of an expression, and will not affect the solution to an equation. The associative property allows to rewrite expressions involving addition or multiplication using any grouping.

Example: (addition)

$$(3x+5)+12=3x+(5+12)$$

Example:(multiplication)

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

(For more information on this, please refer to Lesson 86 or pages s.70-s.82 from the Algebra 1 Module 1 workbook you received)

Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

SOLVING INEQUALITIES

When solving inequalities, similar rules apply. As opposed to rules of equality, there are **rules of inequality**.

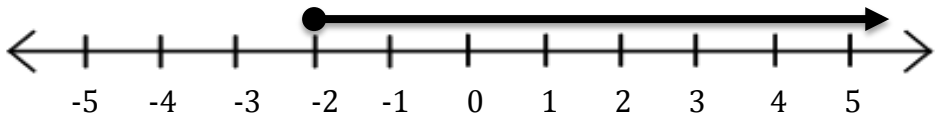
***When finding solutions for inequalities, if you multiply or divide by a negative number, you reverse the inequality sign.

Example: $-3x \leq 15$

$x \geq -5$

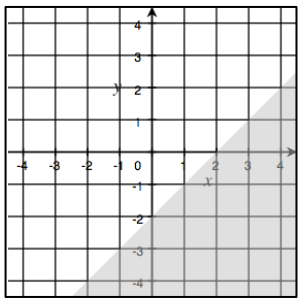
Solutions to inequalities in one variable can be expressed graphically on a number line.

Example: $x \geq -2$



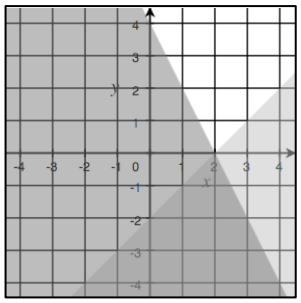
Solutions to linear inequalities in two variables are expressed graphically as half planes on a coordinate plane.

Example: $y \leq x - 2$



Solutions to systems of linear inequalities are expressed as the area that the individual linear inequalities have in common.

Example: $\begin{cases} y \leq x - 2 \\ 2x + y < -4 \end{cases}$



(For more information on this, please refer to Lessons 87 - 91 or pages s.83-104 and s.137-s.150 from the Algebra 1 Module 1 workbook you received)

Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

SOLVING MORE COMPLICATED EQUATIONS

<u>The Zero Product Property:</u> If $ab = 0$, then either $a = 0$ or $b = 0$ or $a = b = 0$.

<u>Examples</u>:

$$(x-3)(2x-8) = 0$$

Either $x - 3 = 0$ *or* $2x - 8 = 0$

When solving equations with **variable expressions in the denominator,** remember to create compound equations to exclude solutions that lead to a denominator of 0.

<u>Example</u>:

$$\frac{4x-12}{x-3} = 4$$

The solution set for this equation would be $\{x\ real \mid x \neq 3\}$

(For more information on this, please refer to Lessons 92 and 93 or pages s.105-s.115 from the Algebra 1 Module 1 workbook you received)

Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

PROBLEM SET

Please complete all questions by Thursday May 7. You should try to complete these problems on your own, but answers will be posted on http://mrrogove.weebly.com

Solve these equations / inequalities for x. Write an explanation for EACH step and provide an answer in set notation.

$-9(3x+2)-15=3$

$-27x-18-15=3$	Distributive Property
$-27x-33=3$	Collect like terms
$-27x-33+33=3+33$	Prop of equality for addition
$-27x=36$	Collect like terms
$-\frac{1}{27}(-27x)=-\frac{1}{27}(36)$	Prop of equality for multiplication
$x=-\frac{4}{3}$	Simplify

$2a+6x=9x-5b$

$2a+6x-2a=9x-5b-2a$	Subtraction Property of equality
$6x=9x-5b-2a$	Collect like terms
$6x-9x=9x-5b-2a-9x$	Subtraction Property of equality
$-3x=-5b-2a$	Collect like terms
$\frac{-3x}{-3}=\frac{-5b-2a}{-3}$	Division Property of equality
$x=\frac{5b+2a}{3}$	Simplify

$5(2x-4)-11=4+3x$

$10x-20-11=4+3x$	Distributive Property
$10x-31=4+3x$	Collect like terms
$10x-31-3x=4+3x-3x$	Property of equality for subtraction
$7x-31=4$	Collect like terms
$7x-31+31=4+31$	Property of equality for addition
$7x=35$	Collect like terms
$\frac{7x}{7}=\frac{35}{7}$	Property of equality for division
$x=5$	simplify

$2(7-4x)<-18$

$14-8x<-18$	Distributive Property
$14-8x-14<-18-14$	Subtraction prop of inequality
$-8x<-32$	Collect like terms
$\frac{-8x}{-8}>\frac{-32}{-8}$	Division property of inequality (switch inequality signs)
$x>4$	Simplify

Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

Consider the following moves and write a sentence explaining why (or why not) equality is maintained? **Do not make any judgments on the sensibilities of performing the particular move…just make a determination of equality.**

Equation: $$\frac{x}{3} = \frac{12}{9}$$ Move: Square both sides. $$\frac{x^2}{9} = \frac{144}{81}$$ Equality is not maintained….there is an extra solution that has been added. Prior to squaring both sides, the solution set included only 4, but after squaring both sides, the solution set became $\{4, -4\}$ which does not work with the original equation.	Equation: $$(2x-3)(x+5)+7(2x-3) = 12$$ Move: Factor out the linear term. $$(2x-3)(7x+35) = 12$$ Equality is not maintained. It's not that you can't factor out the linear term, but the result above is not correct. The correct next line would be: $$(2x-3)(x+5+7) = 12$$ We don't distribute the 7 to the linear term $x+5$, we add it.
Equation: $$\frac{3}{4}(x+2) = \frac{2}{3}(3x-1)$$ Move: Multiply both sides by 12. $$9(x+2) = 8(3x-1)$$ This maintains equality because of the property of equality for multiplication. This process of clearing the fraction is only one way you might begin solving an equation like this.	Equation: $$x+4 = 3x+2$$ Move: Add $x+2$ to both sides. $$2x+6 = 5x+4$$ This does NOT maintain equality. If the move made the equation $2x+6 = 4x+4$ as I intended, it would have maintained equality…this was obviously a typo…OOPS I'm not sure why you would ever want to solve an equation using this move, but should you decide to, you may because of the property of equality for addition. Your solution set does not change as a result of this move.

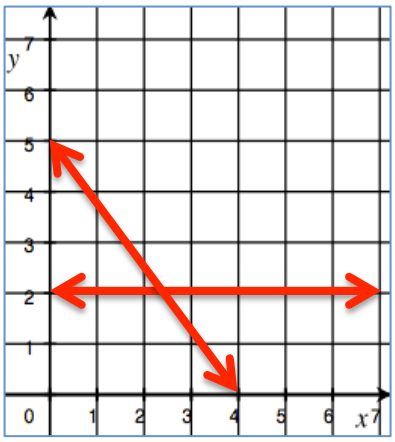
Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

Mr. Rogove wants to get into shape. He plans to burn 2000 calories every week using a combination of the elliptical and the bike machine. The elliptical burns 500 calories an hour, and the bike machine burns 400 calories an hour. Write an inequality that represents the number of hours Mr. Rogove needs to spend on the elliptical and the on the bike machine in a week to burn at least 2000 calories. Let's say he decides that he will be on the bike machine exactly 2 hours a week, show this on the graph, and answer the questions below.

Let x = elliptical and
Let y = bike machine

$$500x + 400y \geq 2000$$
$$y = 2$$



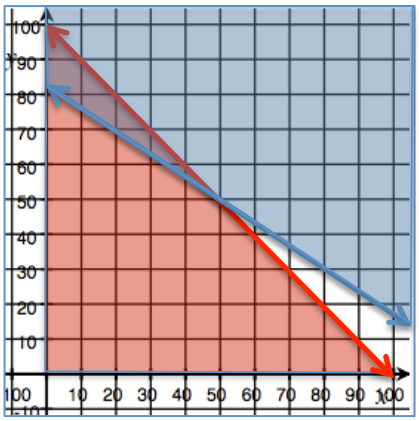
What is the least amount of time Mr. Rogove can spend on the elliptical and still meet his goal? 2 hours and 24 minutes

If Mr. Rogove wants to burn exactly 2500 calories, how many hours does he need to spend on the elliptical? 3 hours and 24 minutes

Jessica won a gift certificate for up to 100 free floor tiles. She plans to use the certificate to remodel her bathroom. She can choose from blue tiles which cover 30 square centimeters and purple tiles which can cover 20 square centimeters. She isn't willing to purchase any additional tiles and needs at least 2500 square centimeters to cover her bathroom floor. Write a system of inequalities and graph a solution that will help Jessica figure out how many of each color tile she should get to cover her floor.

Let x = purple tiles and
Let y = blue tiles

$$x + y \leq 100$$
$$20x + 30y \geq 2500$$



Can Jessica get 50 of each tile if she wants? Yes, this will be exactly 2,500 square centimeters

Would she be able to get 40 purple and 60 blue tiles? Yes, this will be 2,600 square centimeters

Name: **ANSWER KEY** Math 7.2, Period _____

Mr. Rogove Date: _____

Solve each equation below for x and answer the questions.

$(x-3)(2x+5)=(x-3)(x-7)$ $(x-3)(2x+5)-(x-3)(x-7)=0$ $(x-3)[(2x+5)-(x-7)]=0$ $(x-3)(x+12)=0$ $(x-3)=0\ or\ (x+12)=0$ $x=3\ or\ x=-12$ $\{3,-12\}$ Why should you NOT simply divide each side of the equation by $x-3$? You would miss part of the solution set—specifically that -3 could be an answer.	$x^2-36=7(x+6)$ $(x+6)(x-6)=7(x+6)$ $(x+6)(x-6)-7(x+6)=0$ $(x+6)[(x-6)-7]=0$ $(x+6)(x-13)=0$ $(x+6)=0\ or\ (x-13)=0$ $x=-6\ or\ x=13$ $\{-6,13\}$ What linear factor should you NOT divide by to solve this equation? I would not divide by $(x+6)$ because then you would miss part of the solution—specifically that -6 could be a part of the solution.
Create your own equation that has -2 and 5 as its only solutions. Answers will vary…but I suspect many will say the following: $(x+2)(x-5)=0$	A string 60 inches long is to be laid out on a table top to make a rectangle of perimeter 60 inches. Write the width of the rectangle as $15+x$ inches. What is the expression for the length? $15-x$ What is the expression for its area? $(15+x)(15-x)$ What value of x gives an area of the largest possible value? when $x=0$ What kind of rectangle is this? It's a square